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## **Estimating Potential Output and Other Unobservable Variables Using Monetary Policy Transmission Model (the Case of Russia)**

### **Introduction<sup>2</sup>**

Preparations to introduce inflation targeting regime (IT) are being made in the Bank of Russia, including efforts to build a forecasting and monetary policy analysis system and specifically, a monetary policy transmission model as its key element. The model is an ad hoc dynamic stochastic general equilibrium (DSGE) one. Similar models play a role of working horses in many inflation targeting central banks in their decision-making processes.

In these models macroeconomic variables are represented as a difference between their actual and equilibrium levels, i.e. as gaps. Such an approach is based on a treatment, generally adopted by modern macroeconomics, of equilibrium levels as inflation neutral, i.e. non-accelerating or non-decelerating inflation relative to its target level. For instance, if inflation is above its target, that can be viewed as a result of excess demand measured by positive output gap, which in turn can be an outcome of loose monetary conditions (when interest and exchange rates<sup>3</sup> are below their equilibrium levels).

But, while inflation target is explicitly set by monetary authorities, other macroeconomic variables' equilibrium levels are unobservable and hence are subject to estimation. Thus estimation procedures become crucial as unobservable variables paths play a key role in setting a model's dynamics which is essential for making projections and thus — for decision-making process in a central bank.

Within DSGE models unobservables' paths are estimated by multivariate Kalman filter techniques, and they are model-dependent. At a forecasting horizon the equilibrium paths depend also on exogenously set steady-state parameters, to which an economy converges theoretically and by the model set-up. Thus these parameters are very important for forecasts, but their choice is a separate problem which won't be covered here.

The paper presents estimates of key macroeconomic unobservable variables for Russia in 2000 – 2007 within a preliminary version of Russian monetary transmission model. The chosen period covers more or less homogeneous processes in the economy and its choice was also driven by data availability.

The paper's structure is as follows. Section 1 discusses specific features of the model as they are crucial for results of assessing unobservable variables. Section 2 studies whether the estimated unobservables' paths are consistent with Russian economic developments in 2000 – 2007. Section 3 provides a comparison of the model's unobservables' assessments with those obtained by using other methods.

The general conclusion is made regarding consistency of the model unobservables' estimates for making projections.

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<sup>1</sup> Bank of Russia.

<sup>2</sup> Arguments and conclusions presented in the paper reflect the authors' own opinion and do not necessarily represent the point of view of the Bank of Russia.

<sup>3</sup> Measured as price of a unit of national currency.

## 1. Brief description of the Quarterly Projection Model (QPM) of the Bank of Russia<sup>1</sup>

The model is being built in Research and Information Department of the Bank of Russia since 2007 with the IMF technical assistance. Its basic element is a so called small ad hoc transmission model based on New Keynesian theoretical backgrounds. Such models are widely used by inflation targeting central banks. The essence of the model is represented by four behavioral equations describing:

- aggregate demand (IS curve);
- aggregate supply (expectations-augmented New Keynesian Phillips curve);
- arbitrage condition in an open economy (uncovered interest parity condition);
- monetary policy rule (reaction function).

The outstanding feature of this kind of models is the presence of the reaction function which reflects the monetary policy active role in changing monetary conditions in an open economy to affect aggregate demand and supply in order to bring inflation to target. Therefore main efforts in building the country-specific version of a baseline standard model have been concentrated on capturing specific features of Russian transmission mechanism. Its ongoing development accompanied by structural changes in the economy makes the task even more complicated.

### 1.1. Monetary policy transmission mechanism (TM)

The QPM reflects main monetary transmission channels. In the traditional classification there are four of those: exchange rate, interest rate, credit and expectations channels.

Interest rate channel is represented by a link between interbank rates (levels of which are influenced by monetary authorities actions) and bank loan rates which affect aggregate demand stance reflected by output gap.

Exchange rate channel describes a direct exchange-rate pass-through on inflation (via imported goods price inflation), as well as indirect effects of changes in an exchange rate on aggregate demand.

Expectations channel is reflected by lags and leads in each of the behavioural equations.

Credit channel, which is tightly interconnected with the interest rate one, isn't formally included into the model. Though a quantity variable (i.e. money or credit aggregates) is absent, there is an implicit linkage between nominal monetary and macroeconomic variables. It's provided by an interest rate, which represents the cost of money and, as implied by theory, determines quantity of money in the economy. Nowadays the link between the interest rate and quantity of money is rather weak in Russia, but the evidence of growing importance of the interest rate channel, along with real interest rates becoming positive, will ensure an increasing role of the link. In general we believe that monetary conditions in Russian economy are reasonably well described by exchange and interest rates.

In recent years the dominant TM channel in Russia was the exchange rate one, reducing the role of interest rate channel which is usually considered to play the major role within the New Keynesian TM modelling framework. In contrast to the traditional mechanism where changes in exchange rate are induced by interest rates in Russian economy the process is reversed – interest rates were affected by the exchange rate dynamics under the regime of managed floating of ruble relative to a bicurrency basket (since 2005; to US\$ – before)<sup>2</sup>.

<sup>1</sup> Preliminary version.

<sup>2</sup> “The Bank of Russia in 2006 continued to use the ruble value of the bicurrency basket, comprised of the dollar and the euro, as an operational benchmark when conducting currency interventions. This allowed it to flexibly and sensibly respond

The policy was aimed at restraining real ruble appreciation driven by fundamental factors in order to support domestic competitiveness<sup>1</sup>. The main of those fundamentals were a constant rise in world prices of commodities (primarily oil) which constitute a major part of Russian exports, as well as increase in productivity in the country's tradable sector exceeding that in trade partners (Balassa-Samuelson effect)<sup>2</sup>. Restraining real ruble appreciation was achieved through controlling nominal exchange rate movements in presence of price rigidities. This led to short-term slowing down in real appreciation rates and, equivalently, to additional, "unearned" nominal revenues by commodities exporters. Thus an additional nominal demand was created which was not compatible with the economy's capacity to match it by increase in production, which led to accelerating inflation<sup>3</sup>. This in turn resulted in speeding up real appreciation rate to the fundamental level.

Within the framework of the tough managed floating regime, there was little room for conducting monetary policy by changing interest rates. For instance, increase in interest rates would mean widening interest-rates differential and thus an undesirable capital inflow (with capital flows being increasingly deregulated). One more factor of the cautious approach to managing interest rates was a desire to maintain the banking sector stability. Besides, until recently the monetary policy operational procedure was based on regulating base money and foreign exchange market interventions aimed at maintaining stable nominal value of a bicurrency basket, which per se made an effective interest rate policy impossible. It should be also mentioned that a weakness of interest rate transmission channel was inevitable in a situation of underdeveloped financial markets in Russia and their high segmentation. Thus the Bank of Russia had to use a set of instruments with distinct rate for each rather than a universal one which meant an absence of a base rate.

In general, given the policy of rather tough managed floating and liberalising capital flows completed by the middle of 2006, it was practically impossible to conduct monetary policy, at least the one aimed at bringing inflation to target and anchoring inflation expectations in the medium and long-run<sup>4</sup>.

Nevertheless positive current account balance has been steadily decreasing, capital inflows have been moderating since the second half of 2007, hence pressures towards ruble appreciation have been weakening. So conducting an independent interest rates policy is becoming more and more feasible. Ongoing changes in forming market liquidity signal about a growing importance of the domestic sources of the banking system's refinancing and consequently, of the Bank's of Russia lending rates, first of all, of a REPO rate.

These developments as well as the Bank's intention to gradually proceed to a more free floating (and, respectively, to diminish interventions at the forex market) and thus to managing interest rates, allowed us to specify an interest rate as a policy instrument in monetary policy rule. The choice of a reaction function (i.e. monetary policy rule) was one of the most essential problems in building the model. A compromise should be achieved between the *current* and a *future* stance of

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to the euro/dollar fluctuations in implementing its exchange rate policy, and to cushion the fluctuations in the nominal effective rate of the ruble." The Bank of Russia. Annual 2006 Report., p.57. — [http://www.cbr.ru/eng/today/annual\\_report/ar\\_2006\\_en.pdf](http://www.cbr.ru/eng/today/annual_report/ar_2006_en.pdf)

<sup>1</sup> "The main task for the Bank of Russia in containing the ruble's growth caused by fundamental economic factors was to let domestic producers adapt to the changing conditions of competition." — The Bank of Russia. Annual 2006 Report., p.57 — [http://www.cbr.ru/eng/today/annual\\_report/ar\\_2006\\_en.pdf](http://www.cbr.ru/eng/today/annual_report/ar_2006_en.pdf)

<sup>2</sup> For assessments of factors of ruble real equilibrium appreciation see: K.A. Sosunov, A.V. Shumilov. Assessing Russian ruble exchange rate. // Economics Journal of HSE. Vol.9 №2, 2005; Spatafora N., Stavrev E. The Equilibrium Real Exchange Rate in a Commodity Exporting Country: The Case of Russia. // IMF Working Paper No. 03/93

<sup>3</sup> In terms of money supply the same process can be described as an injection of excess liquidity to the economy via foreign exchange interventions by the Bank of Russia in order to constrain nominal ruble appreciation.

<sup>4</sup> This is a well known in international economic hypothesis if 'impossible trinity' saying that it is impossible to have all three of the following at the same time: a fixed exchange rate, free capital movement and an independent monetary policy; a country must pick two out of three.

the transmission mechanism in Russia. According to international practices an interbank interest rate has been chosen as a policy rate in the current version of the model, presuming that it is (or will be) being affected by the Bank of Russia's operations. This choice was also affected by an absence of a single base rate of the Bank.

Exchange rate in the model is represented by the bicurrency basket ruble exchange rate (composed of US\$ with the current share of 0.55 and euro) which plays the role of the operational target of the Bank's of Russia exchange rate policy.

Along with the monetary policy rule specification, essential features of Russian TM and economy were captured by the model's parametrization<sup>1</sup>. Among them are the following:

- moderate (not aggressive) monetary policy reaction to deviations of a projected inflation from target and smoothing out the dynamics of interest rates;
- weak linkages between policy (here — money market) interest rate and banking system interest rates;
- high inertia in economic processes (GDP dynamics, inflation) responsible for a long period of convergence to a steady state;
- low interest-rates elasticity of output;
- higher exchange-rate elasticity of output;
- moderate exchange rate to inflation pass-through.

In general, the essential part of the model can be summarized by four equations supplemented with necessary identities and exogenous processes.

## 1.2. Equations

### 1.2.1. Basic equations

Phillips curve:

$$\pi_t = \alpha_1 \cdot \pi_{t+4} + \alpha_2 \cdot \pi_{t-1} + (1 - \alpha_1 - \alpha_2) \cdot (\Delta l_{S_t} + \pi_t^{for} - \Delta l_{z_t}^{eq}) + \alpha_3 \cdot y_t^{gap} + \varepsilon_t^{(1)}.$$

IS curve:

$$y_t^{gap} = \beta_1 \cdot y_{t+1}^{gap} + \beta_2 \cdot y_{t-1}^{gap} - \beta_3 \cdot (r_{t-1}^{market} - r_{t-1}^{market eq}) + \beta_4 \cdot l_{z_{t-1}}^{gap} + \beta_5 \cdot y_t^{gap for} + \varepsilon_t^{(2)}.$$

Uncovered interest parity condition:

$$l_{S_t} = \delta_1 \cdot l_{S_{t+1}} + (1 - \delta_1) \cdot (l_{S_{t-1}} + \Delta_2 l_{S_t}^{eq} / 4) - (r_{S_t} - r_{S_t}^{for} - \rho_t) / 4 + \varepsilon_t^{(3)}.$$

Monetary policy rule:

$$r_{S_t} = \gamma_1 \cdot r_{S_{t-1}} + (1 - \gamma_1) \cdot (r_{S_t}^{neutral} + \gamma_2 \cdot (\pi_{t+4} - \pi_{t+4}^{tar}) + \gamma_3 \cdot y_t^{gap}) + \varepsilon_t^{(4)}.$$

### 1.2.2. Exogenous processes

Inflation target:

$$\pi_t^{tar} = \pi_{t-1}^{tar} + \eta_t^{(1)}.$$

<sup>1</sup> According to international practices calibration was used to assign parameters to the model. This method is considered to be the best choice at the current stage of the model building, taking into account a limited ability to use statistical and econometric methods coming from short time series, active structural changes in the economy, evolution on monetary policy regime. Besides, certain parameters are not subject to estimation. Estimates derived from VAR-models of Russian monetary transmission and from other empirical approaches were taken into consideration.

Equilibrium output growth:

$$\Delta y_t^{eq} = \chi \cdot \Delta y_{t-1}^{eq} + (1 - \chi) \cdot \Delta y_{ss}^{eq} + \eta_t^{(2)}.$$

Nominal interest rate on banks loans:

$$rs_t^{market} = \xi \cdot rs_{t-1}^{market} + (1 - \xi) \cdot (rs_t + rs_{t+1} + rs_{t+2} + rs_{t+3})/4 + \rho_t^{market} + \eta_t^{(3)}.$$

Real equilibrium interest rate on banks loans:

$$rr_t^{market eq} = \zeta_2 \cdot rr_{t-1}^{market eq} + (1 - \zeta_2) \cdot rr_{ss}^{market eq} + \eta_t^{(4)}.$$

Term premium:

$$\rho_t^{market} = \kappa \cdot \rho_{t-1}^{market} + (1 - \kappa) \cdot \rho_{ss}^{market} + \eta_t^{(5)}.$$

Real equilibrium exchange rate depreciation:

$$\Delta lz_t^{eq} = \zeta_1 \cdot \Delta lz_{t-1}^{eq} + (1 - \zeta_1) \cdot \Delta lz_{ss}^{eq} + \eta_t^{(6)}.$$

Risk premium:

$$\rho_t = \zeta_3 \cdot \rho_{t-1} + (1 - \zeta_3) \cdot \rho_{ss} + \eta_t^{(7)}.$$

Foreign output gap:

$$y_t^{gap for} = 0.85 \cdot y_{t-1}^{gap for} + \eta_t^{(8)}.$$

Foreign nominal interest rate:

$$rs_t^{for} = 0.85 \cdot rs_{t-1}^{for} + \eta_t^{(9)}.$$

Foreign inflation:

$$\pi_t^{for} = 0.85 \cdot \pi_{t-1}^{for} + \eta_t^{(10)}.$$

### 1.2.3. Identities

Year-on year inflation:

$$\pi 4_t = (\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3})/4.$$

Output:

$$y_t = y_t^{eq} + y_t^{gap}.$$

Equilibrium output:

$$y_t^{eq} = y_{t-1}^{eq} + \Delta y_t^{eq} / 4.$$

Nominal policy neutral interest rate:

$$rs_t^{neutral} = rr_t^{market eq} - \rho_t^{market} + \pi 4_{t+4}.$$

Real policy interest rate:

$$rr_t = rS_t - \pi_{t+1}.$$

Real interest rate on banks loans:

$$rr_t^{\text{market}} = rS_t^{\text{market}} - \pi_{t+4}.$$

Nominal exchange rate depreciation:

$$\Delta lS_t = 4 \cdot (lS_t - lS_{t-1}).$$

Nominal equilibrium exchange rate depreciation in 2 quarters:

$$\Delta_2 lS_t^{\text{eq}} = \pi_t + \pi_{t+1} - \pi_t^{\text{for}} - \pi_{t+1}^{\text{for}} + 2 \cdot \Delta lZ_t^{\text{eq}}.$$

Real exchange rate:

$$lz_t = lz_{t-1} + (\Delta lS_t - \pi_t + \pi_t^{\text{for}}) / 4.$$

Real equilibrium exchange rate:

$$lz_t^{\text{eq}} = lz_{t-1}^{\text{eq}} + \Delta lZ_t^{\text{eq}} / 4.$$

Real exchange rate gap:

$$lz_t^{\text{gap}} = lz_t - lz_t^{\text{eq}}.$$

Foreign real interest rate:

$$rr_t^{\text{for}} = rS_t^{\text{for}} - \pi_{t+1}^{\text{for}}.$$

#### 1.2.4. Notations

- $x_t$  – value of domestic variable  $x$  in period  $t$
- $x_{ss}$  – steady state value of  $x$
- $x_t^{\text{eq}}$  – equilibrium value of  $x$  in period  $t$
- $\Delta x_t$  – annualized % change in  $x$  since time period  $t-1$  to  $t$
- $x_t^{\text{gap}}$  – difference between actual and equilibrium values of  $x$  in period  $t$
- $x_t^{\text{for}}$  – value of a foreign variable  $x$  in period  $t$

Basic country-specific parameters of the model are given in Table 1.

Paths of potential output, real equilibrium ruble exchange rate to bicurrency basket, real interest rate on loans were obtained within this model framework. Besides, for the period of 2000 – 2005 a target path was also estimated by Kalman filter as an unobservable. The reasons were as follows.

De-jure the government and the Bank of Russia set an inflation goal for each forthcoming year. It's important to stress that under tight managing of an exchange rate its level is actually predetermined by changes in real equilibrium exchange rate and inflation in trade partners. But this wasn't taken into account while assigning the goal. More than that, as it was already mentioned de-facto tight managed floating made achievement of inflation target impossible. In such a situation the Bank of Russia made hard efforts to resolve this difficult trade-off between targeting exchange rate level and inflation.

Based on these two circumstances, the historical inflation target path was set as a random walk meaning that de-facto it was absent in the IT framework sense. For the years of 2006 – 2007 inflation goals declared in the relevant annual “Guidelines for the Single State Monetary Policy” were used. This rather artificial design aimed at making the target path to converge smoothly to its future values which are presumed to be set under the modelled IT regime. It’s worth mentioning that inflation target path is crucial for all unobservable variables as well as for making projections because equilibrium values are defined as inflation neutral, i.e. consistent with targeted inflation. Thus one should be careful in interpreting the model’s results for 2006 – 2007 given the above mentioned assumptions.

Table 1: Major parameters of the small ad hoc Russia’s monetary transmission model

Equation	Parameter	Economic sense/ Characteristics of the value adopted for Russia	Value	Interval given by international prac- tices
Phillips curve	$\alpha_1$	Forward-looking component (rationality) of expectations / Moderate	0,45	from 0 to 1
Phillips curve	$\alpha_2$	Inflation inertia / Relatively high	0,45	
Phillips curve	$1 - \alpha_1 - \alpha_2$	Exchange rate elasticity of inflation (pass-through effect) / Low	0,1	
Phillips curve	$\alpha_3$	Output gap elasticity of inflation (effect of demand stance on inflation, sacrifice ratio) / Low	0,23	from 0,25 to 0,5 <sup>1</sup>
IS curve	$\beta_2$	Output gap inertia / Low	0,3	from 0,5 to 0,9
IS curve	$\beta_3$	Effect of monetary conditions (on the part of interest rate) on output gap (excess demand /supply) / Low	0,1	in sum from 0,1 to 0,3
IS curve	$\beta_4$	Effect of monetary conditions (on the part of exchange rate) on output gap (excess demand /supply) / Somewhat higher than that of interest rate	0,2	
Monetary policy rule	$\gamma_1$	Smoothing of policy rate / Moderate	0,4	
Nominal interest rate on bank loans	$\xi$	Loan rate inertia / High	0,7	
	$1 - \xi$	Dependence of loan rate on policy interest rate / Low	0,3	

<sup>1</sup> The intervals are very approximate and might change dependent on a model specification.

### 1.3. Estimates of the key unobservable variables



Figure 1: Implicit inflation target

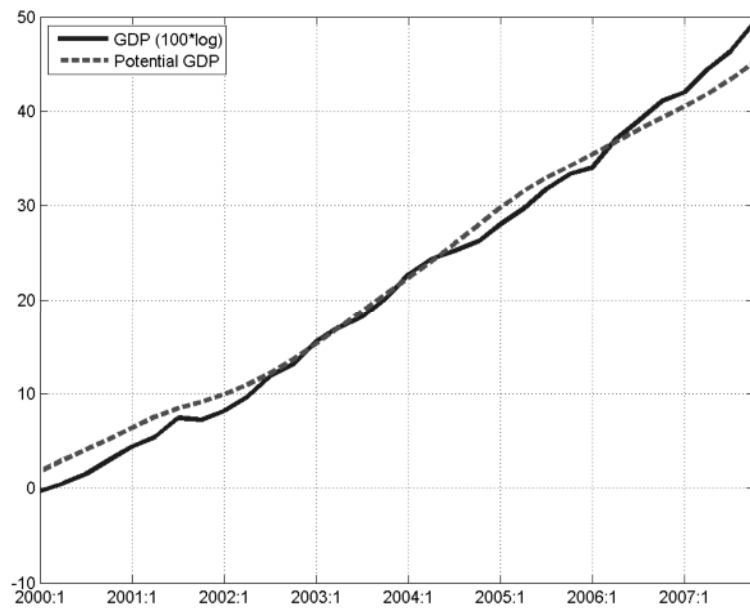


Figure 2a: Actual and potential output

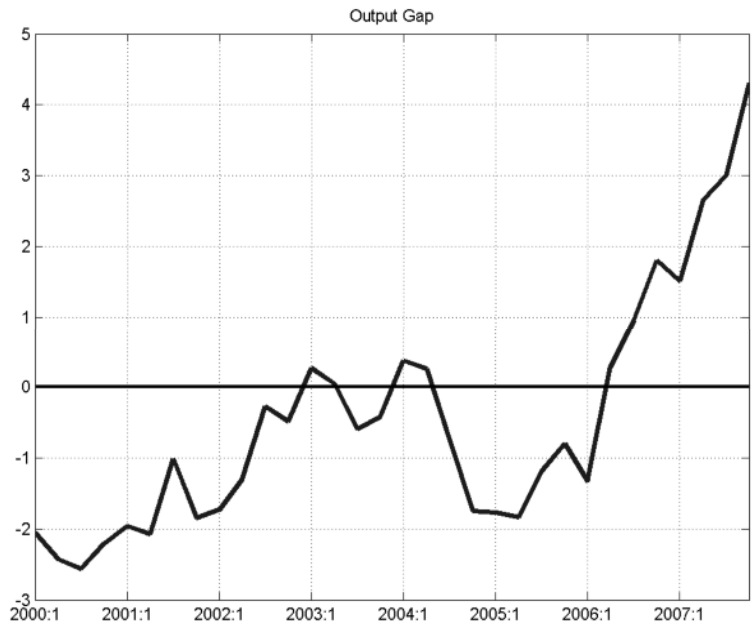


Figure 2b: Output gap

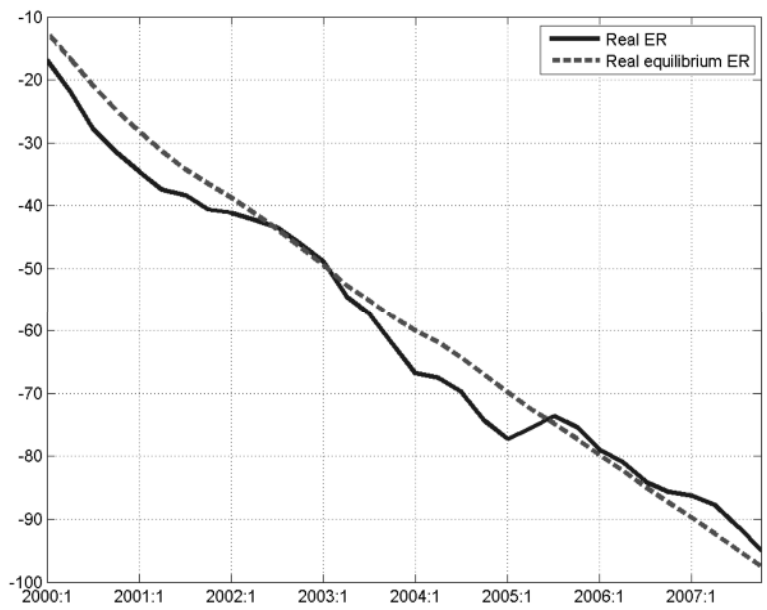


Figure 3a: Actual and equilibrium real ruble exchange rate

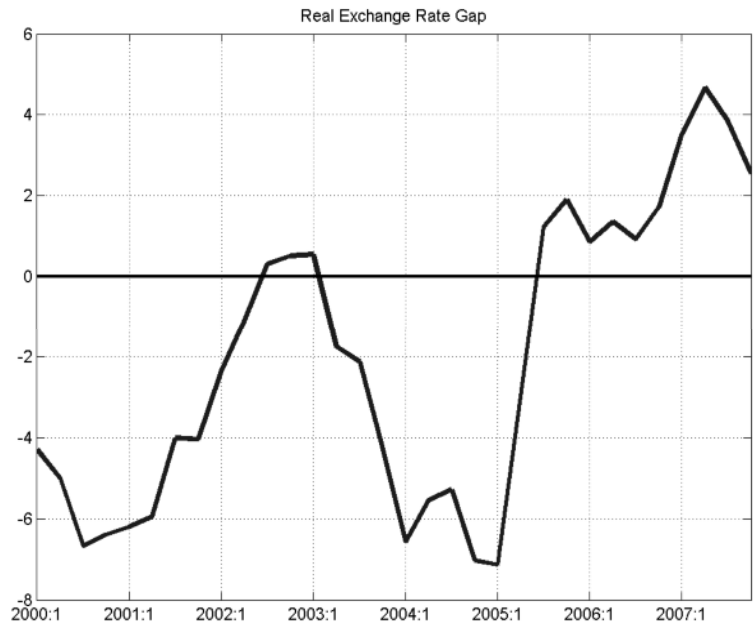


Figure 3b: Real ruble exchange rate gap

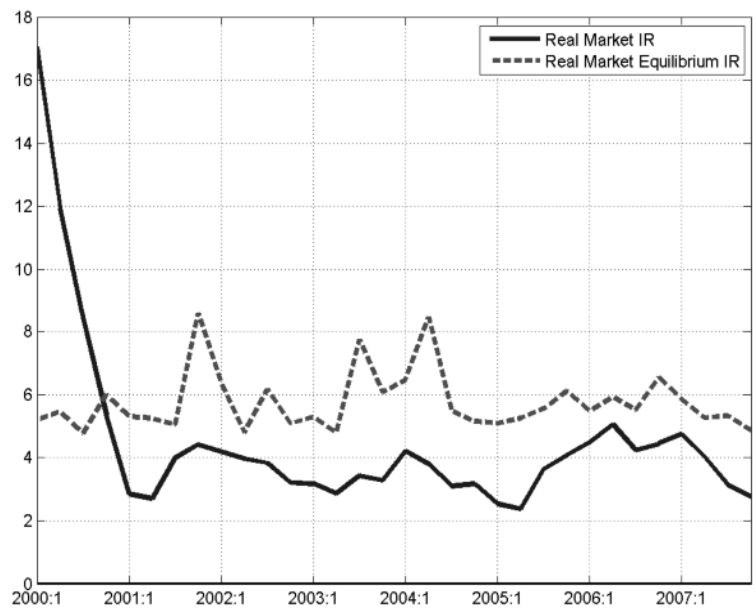


Figure 4a: Actual and equilibrium real bank loans interest rate



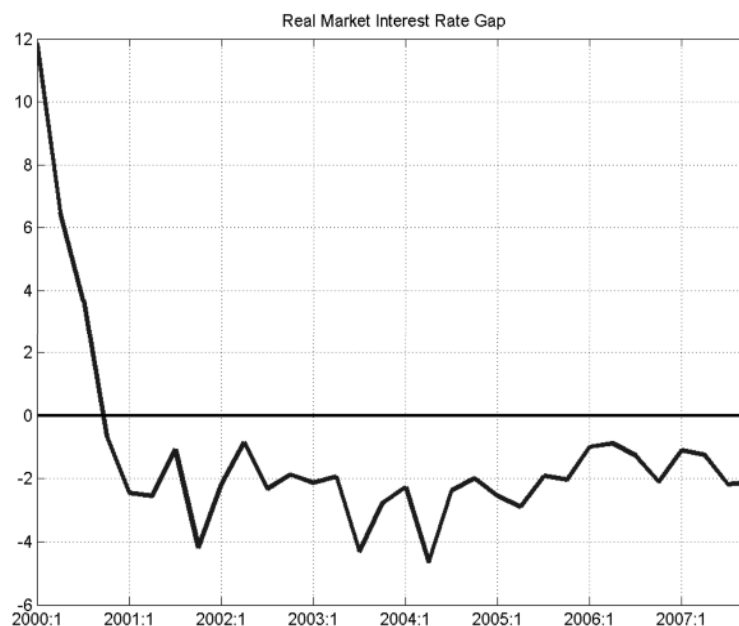


Figure 4b: Real bank loans interest rate gap

## 2. Consistency of the story told by the unobservable variables with stylized facts of Russian economy in 2000-2007

The main means to judge whether unobservables' paths obtained within a model are "right" or acceptable is to check whether a story told by them suits well actual macroeconomic processes and monetary policy stance at a qualitative level.

Nevertheless it's difficult to interpret historical results of the model describing an active role of monetary policy in bringing inflation to target in the Russian case, because IT wasn't in place. Therefore we concentrated our judgements on general macroeconomic story.

The key feature of Russian economic developments after 1998 financial crisis was rather rapid economic growth (at about 7% on average in 1999 – 2007) along with slowing down inflation (from 36.5% in December, 1999 (m-o-m) to 9% in December, 2006). In 2007 price growth accelerated to 11.9%.

To certain extent this acceleration reflected a sharp rise in world food prices. Russia depends on imports of individual food items (see Table 2). Apart from this, the country is one of the world largest grain exporters and thus the domestic market is exposed to the world market developments.

But in 2007 not only consumer prices but also industrial producer prices as well as prices in construction accelerated too. Thus one could speak about general price level upward movement in Russia which was caused not only by external shocks but by domestic inflationary pressures as well.

Our judgement on the reliability of the model's outcomes will be based on their ability to capture these stylized facts of recent Russian economic developments.

### 2.1. Output gap

New Keynesian macroeconomics paradigm supposes that one of the most important tasks of monetary policy is to properly and in time detect a general economic stance, i.e. to what extent output deviates from its long-run trend (or a certain "equilibrium" level). One of the most popular meas-

ures of the economic stance is GDP (or output) gap which is a difference between its actual and potential levels. This indicator is widely used by inflation targeting central banks as a general measure of inflationary pressures in the economy<sup>1</sup>.

Table 2: Imports and exports shares in resources and use of certain food items in 2006, %

	Imports shares in resources of certain food items, %	Exports shares in use of certain food items, %
meat and poultry	39.1	0.0
milk and dairy	18.1	1.3
of which		
butter	47.7	0.4
fat cheeses (including sheep's milk cheese)	41.1	2.2
flour	0.3	2.2
cereals	3.5	13.4
pasta	15.9	10.9
vegetable oils	23.2	22.1
vegetables, water-melon, melon and gourd	9.0	0.4

Source: Rosstat

Predominantly in 2000 – 2006 actual GDP, regardless its high growth rates, didn't exceed its potential level which corresponds to slowing down inflation. Consequently, the potential also stood at a high level which means that production capacities in Russian economy grew intensively. Does this hypothesis match the data?

In 2000-2002 production grew fast driven by active imports substitution (after deep ruble depreciation in 1998) supported by substantial amount of free production capacities. When this source of growth became exhausted (in 2003-2004 when actual output reached the potential one) intensive factors of economic growth started to play a greater role (Table 3). Fixed capital investments increased fast, production modernisation and fixed assets renovation took place, thus creating a basis for productivity growth. Unit costs were diminishing.

But as can be seen from persistent growth in tear and wear of fixed assets, investments were insufficient, and this put limits to non-inflationary growth. Results of surveys<sup>2</sup> show that since 1999 average capacity utilization rate has been growing. Starting from low levels of less than 40% it exceeded 70% on industrial activities and almost reached this level in construction.

In agriculture where capacity constraints (for instance, productivity of soil, heads of livestock) cannot be removed fast, growth rates didn't exceed 3.6%, which is, in particular, much lower than growth in final consumption expenditures by households. It ought to be mentioned that in Russia where living standards of the most part of the population are not high<sup>3</sup>, increase in incomes translates mainly to consumption, of which to a great extent — to consumption of food.

<sup>1</sup> Other indicators of business activity conditions could also be used such as unemployment gap, real marginal costs.

<sup>2</sup> Center of Economic Conjuncture, Government of Russian Federation. Bulletin of business activity in Russia (surveys data). — №3 (31), 2007

<sup>3</sup> In 2007, monthly income of 63.4% of the population was below 12000 rubles, or about US\$ 470. In the 3 quarter of 2007 incomes of 13.3% of Russian citizens were below subsistence minimum.

Table 3: Russian economy indicators in 2000 – 2007.

	2000	2001	2002	2003	2004	2005	2006	2007
GDP, % to previous year	110.0	105.1	104.7	107.3	107.2	106.4	107.4	108.1
Fixed capital investments, % to previous year	117.4	110.0	102.8	112.5	113.7	110.9	116.7	121.1
Expenditures on technological innovations, bln.rub., constant 2000 prices	49.4	52.6	64.1	68.6	66.6	57.1	73.8	...
Putting new fixed assets into operation (without lives-tock), % of fixed assets value, end of period	1.8	2.1	2.2	2.5	2.7	3.0	3.3	...
Costs per 1 ruble of goods and services, %	...	...	89.9	89.6	87.8	87.3	87.7	87.8*
Tear and wear of fixed assets, beginning of period, %	39.4	41.2	44.0	42.2	42.8	44.3	45.4	...
Final consumption expenditures by households, % to previous year	107.3	109.5	108.5	107.5	112.1	111.8	111.3	113.1

Source: Rosstat

\* In January-September

During the period under consideration unemployment decreased from 10.5 to 6.1%.

In general, signs of growing discrepancies between demand and production capacities (despite the growth in the latter) and thus of the economy's overheating became more and more apparent. This is reflected by output gap becoming positive (as for the estimates, in 2006).

## 2.2. Real exchange rate gap

Real equilibrium exchange rate path and deviations of the actual exchange rate from it, i.e. exchange rate gap, within the model framework are consistent with inflation and output gap. Besides, real equilibrium exchange rate path also reflects persistent ruble appreciation based on fundamental factors.

High inflation in 2000 – 2002 as an outcome of 1998 crisis was reflected by high real ruble appreciation. According to the estimates, in real terms ruble appreciated more than at inflation neutral rates (which corresponds to negative real exchange rate gap) which in line with negative output gap promoted fast decline in inflation. In 2003 – 2004 as output was approaching the potential level as a result of exhausting import substitution, real appreciation became more inflation neutral too and in some episodes even pro-inflationary (which can be seen as positive exchange rate gap). In 2004 inflation accelerated. Then in 2004 – 2006 growth in productivity lead to excess supply and thus falling inflation rates. The negative real exchange rate gap corresponded to these developments. Since the end of 2005 inflationary pressures (excess demand) have been building up which resulted in positive output gap and growing ruble undervaluation (the drift of the exchange rate gap to positive area).

## 2.3. Real loans interest rate gap

The model reflects our assumptions about weakness of interest rates channel, on one hand, and low influence on the economy of interest rates on loans to private sector, on the other.

Real interest rates were predominantly negative during the period under consideration and so they could not play a role of an efficient instrument of managing credit activity which implies weakness of interest rates channel. One of the main reasons was a long-lasting structural liquidity surplus in the banking sector. In such a situation it's obvious that bank rates stood at a below-equilibrium level. Nega-

tive interest rates gap according to theory means loose, pro-inflationary monetary policy which in the short run puts upward pressures on output gap. It's important to stress that negative real interest gap is a factor of high availability of credits and a credit boom. For instance, growth in consumer credits and the related increase in consumption strengthened inflationary pressures in 2006 – 2007.

However, this effect wasn't too strong given a still small share of credits in households' consumption financing. This applies also to non-financial enterprises: high profitability in production activities which make a major part of fixed capital investments (mining, transportation, communications) allows them to finance almost a half of the investments by own resources. A share of loans from domestic and foreign banks in 2007 was only 9.4%. Nevertheless its growth from 2.9% in 2000 witnesses increasing availability of the banking financing.

But in general credit conditions didn't put significant limitations to non-inflationary growth during the most part of the period under discussion.

## 2.4. Conclusions

The unobservables' paths estimated within the described framework by Kalman filter could be considered to be appropriate as they allow to produce a story consistent with actual economic developments in Russia in 2000 – 2007.

The most important are the estimates preceding a projection period as they constitute a part of initial conditions. To our mind, the gaps estimates for the end of 2007 match Russian economic situation characterized by signs of overheating, or excess demand, stemming from increase in demand under the constraints from the supply side (positive output gap). It was promoted by loose enough exchange rate policy and low real interest rates.

The general conclusion is that the model could be used for medium-term forecasting under a transition to IT implementation.

Though, to support the inference it would be useful to compare the given above estimates of unobservable variables with those derived from alternative methods.

## 3. Alternative methods of output gap estimation

It was mentioned earlier that for the purposes of the model we define potential output as the level of output that generates no inflationary pressure in a given economy (neither accelerating nor decelerating inflation). In this case one can interpret output gap as a general indicator of demand to supply ratio, in a sense that, other things being equal, a positive gap would mean excess demand and thus stimulation of an increase of prices.

First models that allowed obtaining estimations of the gap in the sense mentioned above were based on purely statistical univariate techniques, without any additional structural assumptions. There are several such techniques, that decompose a time series into a trend and cycle components. In such a decomposition a (stochastic) trend is considered a supply-side component (a potential output): any shocks in its level or slope are permanent, influencing all of its future values. Transitory cyclical deviations of observed values from trend (gaps) correspond to a demand component effects.

### 3.1. Multivariate models based on the Hodrick-Prescott filter

Several approaches to augmenting the filter proposed by Hodrick and Prescott (hereafter – the HP filter) were used to obtain alternative estimations of output gap. Univariate HP filter has several drawbacks. Its basic specification and main points of critique are mentioned in Appendix B. Laxton

and Tetlow (1992) showed that properties of an HP filter can be significantly improved by using additional information about variables that are related to output in New Keynesian economic paradigm, namely unemployment level and inflation. Thus, residuals from the Phillips curve and Okun law can enter the filter's optimization function<sup>1</sup>.

Multivariate filtering was carried out using two different approaches to augmenting the HP filter.

### 3.1.1. Approach 1

The first approach is based on the representation of HP filter in the form of an unobserved components model with common cyclical component for output and unemployment. Univariate specification was expanded with an autoregressive process for the cyclical component of output  $\varphi_t$ ; a similar process was included to describe unemployment. According to Okun law, equilibrium unemployment level  $\mu_t^u$  was specified, while cyclical deviations of unemployment from  $\mu_t^u$  were related to output gap fluctuations:

$$\begin{aligned} y_t &= \mu_t + \varphi_t + \varepsilon_t, & E(\varepsilon_t) &= 0, & \text{var}(\varepsilon_t) &= \sigma_\varepsilon^2 \\ \mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t, & E(\eta_t) &= 0, & \text{var}(\eta_t) &= \sigma_\eta^2 \\ \beta_t &= \beta_{t-1} + \zeta_t, & E(\zeta_t) &= 0, & \text{var}(\zeta_t) &= \sigma_\zeta^2 \\ \varphi_t &= c(L)\varphi_{t-1} + v_t, & E(v_t) &= 0, & \text{var}(v_t) &= \sigma_v^2 \\ u_t &= \mu_t^u + d(L)\varphi_t + \varepsilon_t^u, & E(\varepsilon_t^u) &= 0, & \text{var}(\varepsilon_t^u) &= \sigma_{\varepsilon_u}^2 \\ \mu_t^u &= \mu_{t-1}^u + \beta_{t-1}^u + \eta_t^u, & E(\eta_t^u) &= 0, & \text{var}(\eta_t^u) &= \sigma_{\eta_u}^2 \\ \beta_t^u &= \beta_{t-1}^u + \zeta_t^u, & E(\zeta_t^u) &= 0, & \text{var}(\zeta_t^u) &= \sigma_{\zeta_u}^2. \end{aligned}$$

Maximum number of lags in  $c(L)$  and  $d(L)$  was set to 4 to allow for effects of fluctuations within a single year. Kalman filtering allows to get ML estimations of the parameters of the model, using which it is possible to obtain paths for potential output and equilibrium unemployment.

### 3.1.2. Approach 2

The second approach is to augment the loss function of a standard HP filter. Economic theory suggests that output gap is related to inflation and unemployment dynamics (Laxton, Tetlow, 1992). This information can be formalized to improve filter's estimates by adding the residuals from corresponding equations, expressed in terms of output gap, in the loss function:

$$L = \sum_{t=0}^T (y_t - \mu_t)^2 + \lambda \left( \sum_{t=1}^{T-1} ((\mu_{t+1} - \mu_t) - (\mu_t - \mu_{t-1})) \right)^2 + \alpha_1 \sum_{t=0}^T \varepsilon_t^u + \alpha_2 \sum_{t=0}^T \varepsilon_t^\pi$$

Here  $\varepsilon_t^u$  stand for Okun law residuals,  $\varepsilon_t^\pi$  are residuals from Phillips curve,  $\alpha_1$  and  $\alpha_2$  — coefficients representing significance of unemployment and inflation fluctuations compared to fluctuations of output gap.

<sup>1</sup> Also in some papers (Conway, Hunt 1997; Claus 2000) capacity utilization measure is used for the same purposes.

Okun law is given by:

$$(u_t - \mu_t^u) = k(y_t - \mu_t) + \varepsilon_t^u,$$

where  $u_t$  is the level of unemployment,  $\mu_t^u$  – equilibrium unemployment, estimated by a simple HP filter.

The following equation was estimated to obtain Phillips curve:

$$\pi_t = \pi_t^e + a(L)(y_t - \mu_t) + \varepsilon_t^\pi.$$

Inflation expectations were composed of several lagged levels of inflation and consensus forecast, computed using the data of Development Center<sup>1</sup>; sum of coefficients of these variables was restricted to 1.

### 3.2. Parameterization

To initiate filtration for an unobserved components model, initial values of parameters were needed. Those were chosen so that in the initial period, prior to the first observation, all variables were supposed to be on their potential levels, equal to the values of the first observations.

$\alpha_1$  and  $\alpha_2$  are the coefficients responsible for comparative importance of Phillips curve and Okun law residuals and output variation. Laxton and Tetlow used equal weights for all three. In a later paper by the researchers from the Bank of New Zealand  $\alpha_1$  and  $\alpha_2$  were both set to 2. We also used this approach for such values provide the most sensible estimation of the gap.

Coefficient  $k$  in Okun law equation was set at 0.3, implying approximately 1% decrease in unemployment for every 3% of increase in output. Such value corresponds to modern practice<sup>2</sup>; also it is in line with the estimation derived from unobserved component model (the sum of coefficients  $d_1, \dots, d_4$ , divided by 100 to convert from percentage difference, equals -0.22).

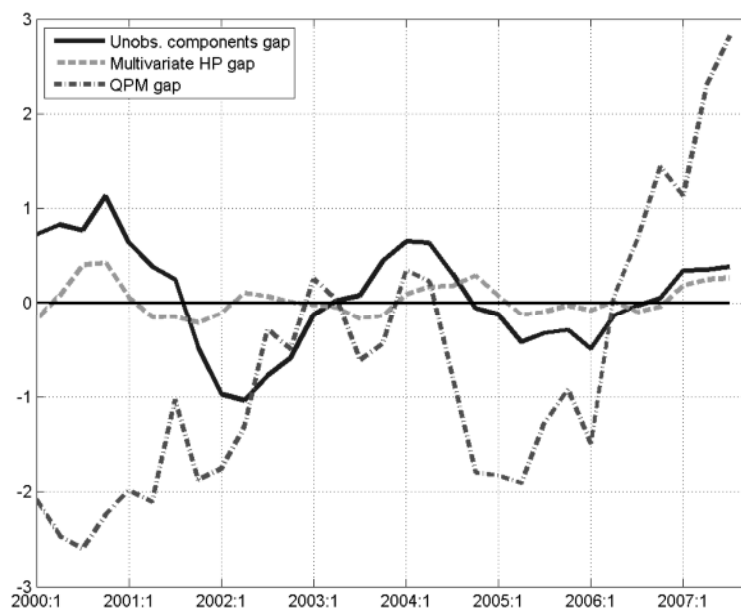
Iterative nature of the estimation of Phillips curve needs further notice. Values of Phillips curve residuals (and thus, overall calculated potential output) depend on the structure of polynomials that enter into the equation. In turn, that structure is estimated on the basis on some initial output gap levels. Phillips curve estimation had thus to be carried out in the following way. On step 1, potential output from a simple HP filter was used to calculate initial values of output gap. Given those estimates, coefficients of gap and inflation expectations polynomials were estimated, using a simple OLS to eliminate autocorrelation in the residuals. Obtained coefficients were used to acquire an expression of Phillips curve residuals in terms of potential and actual output. These expressions, along with expressions for Okun law residuals, were then plugged into augmented PC filter loss function. Then numerical optimization yielded new estimates of potential output, and used to calculate initial values of output gap for step 2. The process continued until there was little change in Phillips curve coefficients between two last steps.

### 3.3. Results and conclusions

The main results are presented on Figure 5. QPM gap estimates are also plotted on that figure for comparison.

<sup>1</sup>Quarterly data of polls are available on the Internet at <http://dcenter.ru>.

<sup>2</sup> Initially an estimate of 0.35–0.4 was obtained by Okun himself (Okun 1962). In the later papers estimates of this coefficient varied for different countries from 0.12 (Japan) to 0.54 (Great Britain) (see Blanchard 2006; Abel, Bernanke 2005).



**Figure 5: Output gap estimates**

Comparing obtained estimates with those of QPM and intuition concerning output dynamics, presented in Section 2.1., it can be seen that the main difference is lower range of fluctuations and a positive values of gap in 2000 – 2001. One of possible explanations for that is unreliability of HP filter based estimates on the ends of sample. Inclusion of additional variables in the model of the filter was supposed to reduce this effect, but obviously, it is impossible to fully solve the problem. At the same time, positive output gap at the beginning of the period corresponds to inflation dynamics. That is not the case for QPM, possibly because an increase of inflation in 2000 is due to factors not related to output; models presented in this section do not include these factors (mainly exchange and interest rate dynamics).

Since the middle of 2001 the obtained estimates generally correspond to intuitive view of output dynamics. Both models, along with QPM, identify the peak of economic activity in 2004 (in the beginning of the year according to QPM and unobserved components models, closer to the end of the year according to multivariate HP estimates), demonstrate the consequent reduction of the gap, which might be explained by an increase in investment growth rates, and finally, another increase of the gap in 2006 – 2007, presumably — due to the surpassing growth of demand given the lack of investments in capital modernization and resource limitations.

Estimates of the gap from the unobserved components model only marginally differ from simple HP filter estimates, but have several significant distinctions from multivariate HP filter. One of the main advantages of the latter is that they better correspond to the macroeconomic definition of the gap — that is to say, they can be more clearly interpreted as a measure of inflationary pressures in the economy. As it can be seen on Figure 6, periods of positive output gap (grey areas) almost perfectly correspond to the periods when (annualized quarterly) inflation rates increased. Thus, the trajectory of the gap obtained with multivariate HP filter might be used as the baseline case for the analysis and interpretation of filtering results of more complicated structural models. It is obvious, though, that the capacity of this model for forecasting and possible monetary policy consequences analysis is severely limited, for survey data on inflation expectations is used in estimation, and there is no way to forecast evolution of such data.

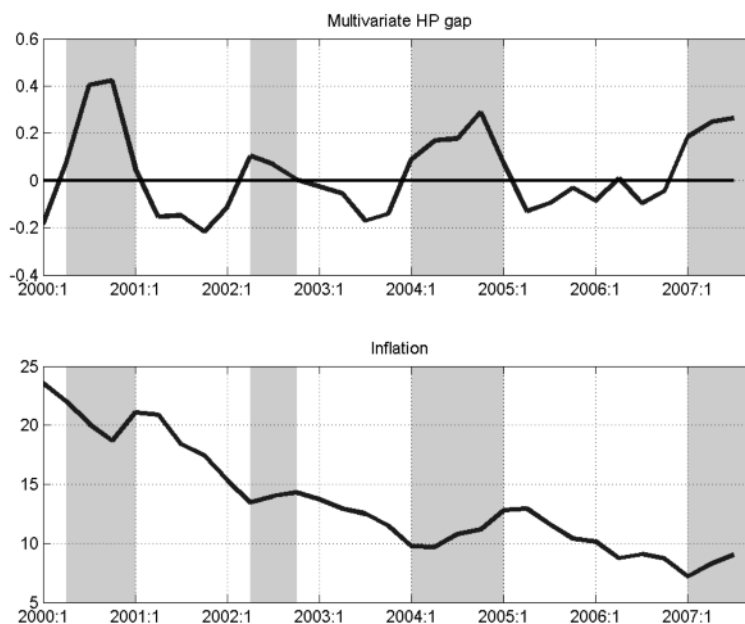


Figure 6: Output gap and inflation dynamics

## Conclusion.

In this paper we described the results of estimation of unobserved variables' trajectories in the monetary policy transmission mechanism model that forms the basis for the ongoing process of the forecasting and policy analysis system development in the Bank of Russia. Dynamics of output gap, equilibrium exchange rate and real interest rate generated by the model is compared to the main tendencies of economic development in Russia in 2000 – 2007.

The results speak in favor of adequacy of QPM estimates and the possibility to use the model in transmission mechanism analysis and supporting decision-making process in monetary policy. It should be pointed out, however, that the estimates of unobserved variables are rather sensitive to parameterization. Therefore we used several methods, based on HP filter, to obtain alternative estimates of output gap. Those estimates qualitatively correspond to QPM for the most part of the period studied; all three estimates confirm an increase in inflationary pressures during the last two years, though they are slightly different in magnitude. At the same time, there are several discrepancies between the estimates, especially at the beginning of the analyzed period. Uncovering the reason for these discrepancies will be the subject of further study.

## Appendix

### A. Technical features of using the Kalman filter to estimate trajectories of unobserved variables in structural models.

As it was mentioned earlier, using the Kalman filter is dictated by the need to combine the economic theory, reflected in the structure of equations of the model of general equilibrium, with expert judgments about the dynamics of unobserved variables as well as with observed statistics.

Kalman filtering allows constructing trajectories for unobserved variables taking into account the structure of the model and statistical data. The results should be viewed as description of the data through the prism of simple non-contradictory statements concerning the principles of the func-

tioning of the economy. Adequacy of such description is checked by comparing the obtained “history” with generally accepted interpretation of actual economic dynamics.

In order to improve the results of filtering (obtain more adequate trajectories) and to avoid situations where Kalman filter is not applicable, it is important to keep in mind various peculiarities of its technical realization. Main problems that arise during the filtering can be divided in two types:

- **structural**, when it is impossible to initiate Kalman filtering — to resolve such issue it is generally required to add, delete or alter some of the model’s equations.
- **parametric**, when filtering can be successfully carried out, but its results do not correspond to reality — that requires more attention to the parameterization of the model.

Below are some common examples, which the authors encountered.

Structural problems arise most often in the beginning of filtering. One example is impossibility of estimation of initial conditions needed to begin the filtering process. Existence of leads in a model is only possible if appropriate number of non-stationary variables is present in the model (Blanchard-Kahn condition). Thus, simple estimation of initial conditions (expected values of the variables and their variances), as in case of stationary models, is often impossible. Solution for this problem is the use of diffuse initial conditions algorithm, when additional filtering procedure is used to estimate initial conditions for non-stationary variables<sup>1</sup>.

The following system is a simple example of impossibility of initial conditions estimation:

$$\begin{cases} \text{lcpi}_t = \pi_t/4 + \text{lcpi}_{t-1}, \\ \pi_t = \pi_{t-1} + \varepsilon_t. \end{cases}$$

If  $\pi$  (growth rate of  $\text{lcpi}$ ) is the only observable variable, then it is obviously impossible to estimate initial value of  $\text{lcpi}$ , for no information about initial price levels can be extracted from growth rates. The problem can be eliminated either by choosing  $\text{lcpi}$  to be observable or, if it is impossible, by excluding this variable from the model and rewriting the equations that contain it terms of growth rates.

Understanding the nature of parametric problems is no less complicated. The most complicated issue here is usually calibration of standard errors of equations’ residuals which characterize deviations of actual data from that calculated in the model. From the filtering point of view, the magnitudes of standard errors of residuals also characterize explanatory power of unobserved variables that are included in a given equation: the more is the residual’s standard error, the less variation is explained by the unobserved variables in the equation.

Obtaining estimates of standard errors of the residuals based on the results of filtering can also contribute to understanding those results. For example, if those estimates exceed a priori values of the standard errors, then explanatory power of corresponding unobserved variables might be overvalued.

## B. Univariate HP filter

Among the techniques commonly used for univariate filtering there are HP and Baxter-King filters, Beveridge-Nelson decomposition, Markov chains based methods and some other<sup>2</sup>. These methods allow to obtain estimates for potential output and output gap, though reliability of the estimates is questioned by many researchers. To review the main points of the critique it is useful to present HP filter in form of a local linear trend model.

<sup>1</sup> Detailed description of the main methods of estimation of diffuse initial conditions can be found in Harvey (1989).

<sup>2</sup> A review of papers on many of those can be found, for example, in Canova 1999.

According to standard definition HP filter is a series that minimizes the loss function:

$$L = \sum_{t=0}^T (y_t - \mu_t)^2 + \lambda \left( \sum_{t=1}^{T-1} ((\mu_{t+1} - \mu_t) - (\mu_t - \mu_{t-1})) \right)^2,$$

where  $y_t$  stands for observed series. Thus for GDP series  $\mu_t$  can be regarded as potential output,  $(y_t - \mu_t)$  – as output gap. In a more general framework, it can be presented as a local linear trend model:

$$\begin{aligned} y_t &= \mu_t + e_t, & E(e_t) &= 0, & \text{var}(e_t) &= \sigma_e^2 \\ \mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t, & E(\eta_t) &= 0, & \text{var}(\eta_t) &= \sigma_\eta^2 \\ \beta_t &= \beta_{t-1} + \zeta_t, & E(\zeta_t) &= 0, & \text{var}(\zeta_t) &= \sigma_\zeta^2. \end{aligned}$$

This is an unobserved components model that can be used to decompose  $y_t$  into “potential output” trend  $\mu_t$  and residual component  $e_t$ . Trend is supposed to be a random walk process with drift  $\beta_t$ , which is, in turn, a random walk itself. HP filter estimates can be presented in form of this model with  $\sigma_\eta^2 = 0$  (Harvey, Jaeger 1993). The only parameter of the filter,  $\lambda$ , is then given by a ratio of two remaining error terms variances<sup>1</sup>:  $\lambda = \frac{\sigma_e^2}{\sigma_\zeta^2}$ .

The model can be further expanded by decomposing residual term into cyclical component  $\varphi_t$ , which would behave according to some intuitive perceptions regarding the gap dynamics, and a completely random residual:

$$y_t = \mu_t + \varphi_t + \varepsilon_t, \quad E(\varepsilon_t) = 0, \quad \text{var}(\varepsilon_t) = \sigma_\varepsilon^2$$

Additional assumption of cyclical behavior of  $\varphi_t$  can be formalized, for example, with an autoregressive process that has some limitations on coefficients and variance of residual (Guay, St-Amant 1996)<sup>2</sup>.

One can obtain potential output estimates by using Kalman filter and maximum likelihood. Also, estimation provides another criterion for HP filter applicability: testing  $\sigma_\eta^2 = 0$ . One of the main directions of criticism is that such a hypothesis is often rejected. Harvey and Jaeger (1993) tested this hypothesis on the GDP data for various countries, and concluded that it holds only for USA, which allowed them to say that HP filter was “tailored” to extract information about business cycles from US GDP.

Another significant drawback of HP filter is the so-called “end point problem”. Due to the fact that the filter is actually a two-sided moving average<sup>3</sup>, estimates of decomposition in current period (the

<sup>1</sup> In his paper about parameterization of the filter Prescott explained his choice of  $\lambda=1600$  for quarterly data by the fact that such value resulted in periodic cycles with the period of 32 quarters, which he considered a good approximation of reality (Prescott, 1986).

<sup>2</sup> Alternatively, one can use a widespread approach of modeling cyclical behavior with stochastic sinusoids (Harvey, 1989):

$$\begin{aligned} \varphi_t &= \rho \cos \lambda_c \varphi_{t-1} + \rho \sin \lambda_c \tilde{\varphi}_{t-1} + \psi_t, & E(\psi_t) &= 0, & \text{var}(\psi_t) &= \sigma_\psi^2 \\ \tilde{\varphi}_t &= -\rho \sin \lambda_c \varphi_{t-1} + \rho \cos \lambda_c \tilde{\varphi}_{t-1} + \tilde{\varphi}_t, & E(\tilde{\varphi}_t) &= 0, & \text{var}(\tilde{\varphi}_t) &= \sigma_\psi^2 \end{aligned}$$

Such form provides clear interpretation for the parameters. Here  $\rho$  determines the amplitude of cycles, and  $\lambda_c$  is frequency of a cycle in radians. Simple HP filter can also be thought of in terms of this model with  $\rho=0$  and/or  $\sigma_\psi^2 = 0$ .

<sup>3</sup> Two-sided in a sense that it uses both past and future values of the observed series to derive the estimates of decomposition.

last observation in a sample) are unreliable in a sense that they are subject to possible significant revisions after new data is added and the estimation is carried out over the extended sample. This flaw reduces the appeal of using HP filter for making monetary policy decisions, for it is the estimation of the current state of economy that is important in that area<sup>1</sup>.

Finally, the results of Monte-Carlo based experiments suggest that the accuracy of decomposition strongly depends upon the characteristics of the data generating process. Guay and St-Amant (1996) showed that the restrictions under which the HP filter with  $\lambda = 1600$  will yield accurate decomposition, can hardly be attributed to actual GNP series.

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<sup>1</sup> Butler (1996) suggested one of the ways to solve the problem — extending the loss function to include a penalty for the deviation of potential output from some fixed long-run growth rate during the last several periods (usually — 12 periods, for Baxter and King showed (Baxter, King 1995) that HP filter is subject to significant end-point revisions during 12 first and 12 last periods.